Mesh Reconnection Method for Solution of Elliptic Problems

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Recently, as part of the Transport Methods project, a code for conservative interpolations on polygonal meshes, focusing on adaptive mesh refinement (AMR), has been developed in T-7. The program first solves an elliptic problem on some 2D mesh, estimates local errors on its edges, then refines the mesh where necessary and computes the problem again. Repeating this for a few times can considerably improve the precision.

One of my tasks for this summer was to implement a set of routines which will adapt the mesh by node reconnection instead of refinement. The most straightforward way to do this is by the so-called edge swapping process on triangular meshes, where we loop through all interior edges and if it is possible and useful (i.e. if it decreases local error of solution without tangling the mesh), we swap the edge in replacing one diagonal of the surrounding quadrilateral patch by the other one.

Lately we developed a family of similar methods [1], but with other focus and application: since our goal was to change the mesh so that some given discrete function can be interpolated on it as precise as possible, the local quality measure was based on minimization of selected terms in Taylor expansion. This time, we have an elliptic solver with its own error estimator derived from the real physical problem being solved. Estimation of the local error requires multiple solution of local elliptic problems using Support Operator Method [2], least-squares fitting of discrete values and numerical integration. The actual mesh reconnection is then done at once in a way that ensures efficiency and does not depend on order in which edges are processed.

These new routines have been incorporated into the mesh adaptation code, which is being further developed by T-7 team members and summer students. As usual, we used the MSTK toolkit to represent and handle the mesh structure, since this is becoming a standard framework for meshing projects in the group.

After extensive testing and comparison with results of a semi-analytical approach (where discrete values of pressure have been computed from the analytical function), we conclude that sufficiently fine meshes reconnected by our method allow us to solve elliptic problems with significantly higher accuracy. We believe, that further improvements will follow after combination with other strategies, like mesh untangling [3] and adaptive mesh refinement [4].

Acknowledgements

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References

[1] P. Váchal, M. Shashkov, and R. Garimella. Rezoning of triangular ALE meshes by node reconnection. Technical Report LA-UR-04-3728, Los Alamos National Laboratory, 2004.

[2] M. Shashkov. *Conservative Finite-Difference Methods on General Grids*. CRC Press, Boca Raton, 1995.

[3] P. Váchal, R. Garimella, and M. Shashkov. Untangling of 2D meshes in ALE simulations. *J. Comp. Phys.*, 196(2):627–644, 2004.

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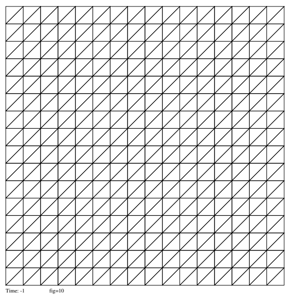
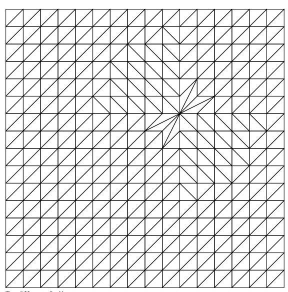
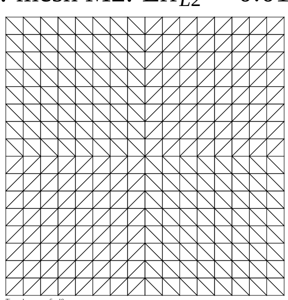
Orig. mesh M1. $\mathrm{Err}_{L2} = 0.01373$



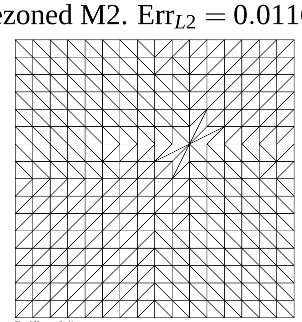
Rezoned M1. $\mathrm{Err}_{L2} = 0.01166$



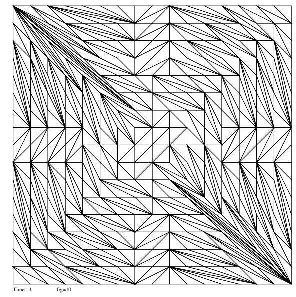
Orig. mesh M2. $\mathrm{Err}_{L2} = 0.01353$



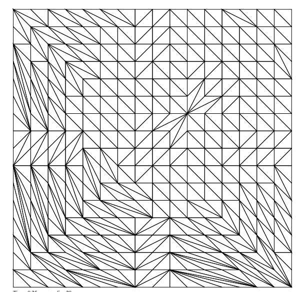
Rezoned M2. $\mathrm{Err}_{L2} = 0.01166$



Orig, mesh M3. $\mathrm{Err}_{L2} = 0.04111$



Rezoned M3. $\mathrm{Err}_{L2} = 0.01169$



Adaptation of three different meshes on domain $[-0.5, 0.5]^2$ *for solution of Dirichlet problem with pressure given by* $p(x,y) = 1 - \tanh\left(|(x,y) - (0.5, 0.5)t|^2/e\right)$, *where* $t = 0.25$ *and* $e = 0.01$. *Global* L_2 *error was decreased by 15, 14, resp. 71%. Note, that swapping was not necessary where the function is flat, i.e. far from the circular region shown below by isolines. Though each of the final meshes looks different, they are almost equally suitable for this problem (*L_2 *error differs mutually by less than 0.2%).*